AFRL-AFOSR-VA-TR-2016-0232

A Mathematical Theory of System Information Flow

Michael Mislove
ADMINISTRATORS OF THE TULANE EDUCATIONAL FUND THE
6823 SAINT CHARLES AVE
NEW ORLEANS, LA 701185665

06/27/2016
Final Report

DISTRIBUTION A: Distribution approved for public release.

Air Force Research Laboratory
AF Office Of Scientific Research (AFOSR)/RTA2

Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
17-06-2016	Final	27MAR2013 - 31MAR2016

4. TITLE AND SUBTITLE

A Mathematical Theory of System Information Flow

5a. CONTRACT NUMBER

5b. GRANT NUMBER

FA9550-13-1-0135

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

Michael Mislove
Tyler Barker
Peter Bierhorst
Department of Mathematics
Tulane University

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

ADMINISTRATORS OF THE TULANE EDUCATIONAL FUND, THE
TULANE UNIVERSITY
6823 SAINT CHARLES AVE NEW ORLEANS LA 70118-5665

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Force Office of Scientific Research
875 North Randolph Street
Suite 325, Room 3112
Arlington VA, 22203

10. SPONSOR/MONITOR'S ACRONYM(S)

US AFOSR

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

No restrictions

13. SUPPLEMENTARY NOTES

14. ABSTRACT

The initial goal of this project was to study information flow in computational systems using techniques from information theory, domain theory and other areas of mathematics and computer science. Over time, the focus shifted toward a better understanding of random variables, in particular from a domain-theoretic perspective. The research focused more narrowly on the relationship between random variables, domain theory and related work on information theory. The results produced by the project include two new models for probabilistic computation. one of which models randomized algorithms and the other of which is suitable for analyzing crypto-protocols. The project also produced a new statistical testing regimen for evaluating empirical tests of quantum phenomena that have since been the basis for validating the first loophole free tests of nonlocality. Finally. the project applied Skorohod's Theorem to analyze the domain structure of certain domains, and conversely, produced a domain-theoretic proof of Skorohod's Theorem that also shows every measure on countably-based coherent domains is the image of Haar measure on the Cantor set

15. SUBJECT TERMS

Classical and quantum information and computation, probabilistic computational models

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
U	U	U	UU		Michael Mislove
					19b. TELEPHONE NUMBER *(Include area code)* (504) 865-5803

Reset

INSTRUCTIONS FOR COMPLETING SF 298

1. REPORT DATE. Full publication date, including day, month, if available. Must cite at least the year and be Year 2000 compliant, e.g. 30-06-1998; xx-06-1998; xx-xx-1998.

2. REPORT TYPE. State the type of report, such as final, technical, interim, memorandum, master's thesis, progress, quarterly, research, special, group study, etc.

3. DATES COVERED. Indicate the time during which the work was performed and the report was written, e.g., Jun 1997 - Jun 1998; 1-10 Jun 1996; May - Nov 1998; Nov 1998.

4. TITLE. Enter title and subtitle with volume number and part number, if applicable. On classified documents, enter the title classification in parentheses.

5a. CONTRACT NUMBER. Enter all contract numbers as they appear in the report, e.g. F33615-86-C-5169.

5b. GRANT NUMBER. Enter all grant numbers as they appear in the report, e.g. AFOSR-82-1234.

5c. PROGRAM ELEMENT NUMBER. Enter all program element numbers as they appear in the report, e.g. 61101A.

5d. PROJECT NUMBER. Enter all project numbers as they appear in the report, e.g. 1F665702D1257; ILIR.

5e. TASK NUMBER. Enter all task numbers as they appear in the report, e.g. 05; RF0330201; T4112.

5f. WORK UNIT NUMBER. Enter all work unit numbers as they appear in the report, e.g. 001; AFAPL30480105.

6. AUTHOR(S). Enter name(s) of person(s) responsible for writing the report, performing the research, or credited with the content of the report. The form of entry is the last name, first name, middle initial, and additional qualifiers separated by commas, e.g. Smith, Richard, J, Jr.

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES). Self-explanatory.

8. PERFORMING ORGANIZATION REPORT NUMBER. Enter all unique alphanumeric report numbers assigned by the performing organization, e.g. BRL-1234; AFWL-TR-85-4017-Vol-21-PT-2.

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES). Enter the name and address of the organization(s) financially responsible for and monitoring the work.

10. SPONSOR/MONITOR'S ACRONYM(S). Enter, if available, e.g. BRL, ARDEC, NADC.

11. SPONSOR/MONITOR'S REPORT NUMBER(S). Enter report number as assigned by the sponsoring/ monitoring agency, if available, e.g. BRL-TR-829; -215.

12. DISTRIBUTION/AVAILABILITY STATEMENT. Use agency-mandated availability statements to indicate the public availability or distribution limitations of the report. If additional limitations/ restrictions or special markings are indicated, follow agency authorization procedures, e.g. RD/FRD, PROPIN, ITAR, etc. Include copyright information.

13. SUPPLEMENTARY NOTES. Enter information not included elsewhere such as: prepared in cooperation with; translation of; report supersedes; old edition number, etc.

14. ABSTRACT. A brief (approximately 200 words) factual summary of the most significant information.

15. SUBJECT TERMS. Key words or phrases identifying major concepts in the report.

16. SECURITY CLASSIFICATION. Enter security classification in accordance with security classification regulations, e.g. U, C, S, etc. If this form contains classified information, stamp classification level on the top and bottom of this page.

17. LIMITATION OF ABSTRACT. This block must be completed to assign a distribution limitation to the abstract. Enter UU (Unclassified Unlimited) or SAR (Same as Report). An entry in this block is necessary if the abstract is to be limited.

AFOSR Final Project Report

Project Title:	A Mathematical Theory of System Information Flow
Award Number:	FA9550-13-1-0135
Start Date:	Mar 27, 2013
Program Manager:	**Dr.** Tristan Nguyen, AFOSR/RTC Phone: (703) 696-7796 FAX: (703) 696-7360 Email: tristan.nguyen@afosr.af.mil
Principal Investigator:	Professor Michael Mislove Departments of Computer Science and Mathematics Tulane University New Orleans, LA 70118 Phone: (504) 865-5803 FAX: (504) 865-5876 Email: mislove@tulane.edu

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
12-06-2016

2. REPORT TYPE
Final

3. DATES COVERED *(From - To)*
27MAR2013 - 31MAR2016

4. TITLE AND SUBTITLE

A Mathematical Theory of System Information Flow

5a. CONTRACT NUMBER

5b. GRANT NUMBER
FA9550-13-1-0135

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

Michael Mislove
Tyler Barker
Peter Bierhorst
Department of Mathematics
Tulane University

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

ADMINISTRATORS OF THE TULANE EDUCATIONAL FUND, THE
TULANE UNIVERSITY
6823 SAINT CHARLES AVE NEW ORLEANS LA 70118-5665

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

AF OFFICE OF SCIENTIFIC RESEARCH
875 NORTH RANDOLPH STREET, RM 3112 ARLINGTON VA 22203

10. SPONSOR/MONITOR'S ACRONYM(S)
US AFOSR

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

No restrictions

13. SUPPLEMENTARY NOTES

14. ABSTRACT

The initial goal of this project was to study information flow in computational systems using techniques from information theory, domain theory and other areas of mathematics and computer science. Over time, the focus shifted toward a better understanding of random variables, in particular from a domain-theoretic perspective. The research focused more narrowly on the relationship between random variables, domain theory and related work on information theory. The results produced by the project include two new models for probabilistic computation. one of which models randomized algorithms and the other of which is suitable for analyzing crypto-protocols. The project also produced a new statistical testing regimen for evaluating empirical tests of quantum phenomena that have since been the basis for validating the first loophole free tests of nonlocality. Finally. the project applied Skorohod's Theorem to analyze the domain structure of certain domains, and conversely, produced a domain-theoretic proof of Skorohod's Theorem that also shows every measure on countably-based coherent domains is the image of Haar measure on the Cantor set

15. SUBJECT TERMS

Classical and quantum information and computation, probabilistic computational models

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
U	U	U	UU		Michael Mislove

19b. TELEPHONE NUMBER *(Include area code)*
(504) 865-5803

Reset

Summary and Principal Results

The initial goal of this project was to study information flow in computational systems using techniques from information theory, domain theory and other areas of mathematics and computer science. The aims were a deeper understanding of information flow that would support computational models to reason about properties of such systems. Over time, the focus shifted — it became clear that a better understanding of random variables, which form the basis for modern treatments of Shannon's Information Theory, was needed, in particular from a domain-theoretic perspective. With that somewhat altered goal, the research became more narrowly focused on the relationship between random variables and domain theory and on related work on information theory. Over the course of the project, several achievements have been recorded:

1. Research on domain random variables and models of computation. The standard domain-theoretic approach to modeling probabilistic choice as a computational effect is to use sub-probability measures on the underlying semantic domain. This approach has known flaws: first, there is no known cartesian closed category of domains that is invariant under the sub-probability measures functor, and second, there is no distributive law of the sub-probability measures functor over any of the monads for nondeterminism. As an alternative, the PI and one of his graduate students have pursued a program of developing monads of sub-probability measures over domains. The idea is to choose for a (sub-)probability space a domain whose family of sub-probability measures is known to be well-behaved, and then to implement probabilistic choice over a semantic domain D using Scott-continuous random variables from the chosen sub-probability space to D. This approach has seen two significant advances over the course of this project:
   - First, Tyler Barker, a PhD student working under the PI's direction devised a monad for randomized algorithms. This monad is based on sub-probability measures on the Cantor tree, the prototypical model for outcomes of coin tosses, implemented by a Scott-continuous map into the semantic domain that carries the meanings of programs. Barker's dissertation includes a proof that this approach produces a monad over the three most important cartesian closed categories of domains: bounded complete domains, retracts of bifinite domains and finitely separated domains. It also shows that this monad enjoys a distributive law over one of the nondeterminism monads. More generally, this construction has been used to give a model of randomized PCF, an idealized toy language that supports the lambda calculus, arithmetic over the natural numbers and randomized algorithms over these data. The monad also generalizes a recent construction of the stochastic lambda calculus by Dana Scott. Barker's monad has been implemented in SML and tested on the Miller-Rabin Primality Testing Algorithm. The construction requires sharing the same coin tosses among participating processes, but it turns out that this makes it more efficient when testing whether one of two numbers is prime, for example.
   - A second monad of random variables has been devised by the PI that overcomes a limitation of the monad for randomized algorithms — the requirement that coin tosses (or whatever source of randomness is used) be shared among component processes. This requirement is not appropriate for applications such as crypto-protocols where honest users need to have independent, secret sources of randomness during their execution of the protocol. This second monad supports this. An interesting aspect emerged during this research. In formulating the monad of random variables, it was realized that the semantic domain in which random variables take their values must support nondeterminism — the reason is

that iterations of the monad could produce different choices for the same sequence of coin tosses when processes are composed, and the semantic domain must necessarily record those choices. This provides an interesting insight into the relationship between probabilistic choice and nondeterminism. While probabilistic choice is a form of nondeterminism, we distinguish it from other forms of nondeterminism, in order to understand the nature of probabilistic choice more clearly.

- This line of research was underpinned by an in-depth investigation of sub-probability measures on the Cantor tree that are supported on antichains. These measures appear to be the prototypical ones that occur in the outcomes of a number of coin flips — the first flip results in heads or tails, and this pair of outcomes forms an antichain when models in the Cantor tree. Likewise, the outcome of n flips of a coin also form an antichain — the n-bit words, where 0 denotes Tails and 1 Heads. These results were part of the mathematical basis of Barker's PhD thesis, and the more expressive model devised in the PI's results about probabilistic choice described above. The research also uses heavily the fact that the Cantor tree consists of the Cantor set as its set of maximal elements, and that, viewed as a domain, the finite quotients of the Cantor set form a family of compacta whose projective limit is the Cantor set. (An alternative view of the same construction is obtained by applying Stone Duality with the Cantor set as a projective limit of finite sets.)
- In related work, the author collaborated with a postdoc working under his direction to produce interesting new results about Cantor groups. Schmidt (the inventor of Schmidt games for badly approximable reals) proved in the 1980s that the canonical map from the Cantor group (qua countable product of two-point groups) to the unit interval sends Haar measure on the Cantor set to Lebesgue measure on the unit interval. The work with the postdoc greatly generalizes this result. We showed that any topological group structure on the Cantor set — abelian or nonabelian(!) — has the same property: there is a canonical map from the Cantor group onto the unit interval and this map sends Haar measure to Lebesgue measure. Moreover, we showed that any two such group structures admit a Borel isomorphism between the underlying measure spaces. These results play a role in the work on Skorohod's Theorem and the theory of domain random variables reported below.

2. Research on the Skorohod's Theorem from a domain perspective produced two results:
    - First, a simple, domain-theoretic proof that the sub-probability measures over any complete chain form a continuous lattice was discovered. This result is remarkable, since the only prior results about the fine domain structure of the sub-probability measures over a domain consisted of the fact that any rooted tree has a bounded complete domain of sub-probability measures, and that the sub-probability measures over a finite reversed tree are a retract of a bifinite domain. The proof is inspired by the standard proof of Skorohod's Theorem in the case that the target Polish space is the unit interval.
    - Second, a domain-theoretic proof of Skorohod's Theorem has been found. It differs from the standard proof of the theorem, in that we use the usual domain-theoretic approach in which measures are approximated by simple measures, and the result for simple measures is proved directly, with the general result following form this. The same proof yields a sharper result for coherent domains with countable bases: it allows us to show that every Borel measure on such a domain is the Scott-continuous image of Haar measure on the Cantor set. This

result is then used to prove Skorohod's Theorem, since any Polish space can be embedded as the set of maximal elements of a countably based, bounded complete domain. In particular, the random variables that are used to realize a measure on a Polish space as the image of Lebesgue measure on the interval is the restriction to the Polish space of a Scott-continuous random variable into the ambient domain in which the Polish space is embedded.

- These results are being written up, so the listing under publications is "in preparation", but this final report would be incomplete if they were not mentioned.

3. Research on statistical tests evaluating empirical results based on Bell's Theorem: Peter Bierhorst, a PhD student working under the direction of the PI developed the first definitive statistical tests for rukling out non-locality as an alternate theory to quantum mechanics. These results support reasoning about experimental results designed to validate that quantum mechanics is a correct theory using empirical results based on Bell's Theorem. Most experimenters use the standard Hoeffding Inequality to determine the statistical significance of their experimental results, but this is a crude estimate, in the sense that it is not tailored for anything special about quantum mechanics or experiments to test it against alternative theories. Peter devised a regime for evaluating empirical tests that rely on the CHSH inequality, and the results obtained are much sharper than prior interpretations. In addition, his approach allows the usual assumption that the trials are i.i.d. to be relaxed — this is also an important advance, since proving i.i.d. is usually quite involved. There are numerous experiments, often using photons, to test Bell's Inequality recorded in the literature, but the results have always been subject to `loopholes" — explanations such as nonlocality or signaling that might explain why the results differ from the classical setting. Peter focused on non-locality as an alternative theory and experiments using the CHSH inequality, and devised a statistical procedure that would produce loophole free tests against non locality if the tests were properly conducted. In his dissertation, Peter demonstrated the utility of his work by analyzing existing empirical data, and showed his approach gives sharper statistical evidence than had been reported in the original research. Upon graduation in August 2014, Peter was awarded a postdoctoral position at NIST-Boulder and has continued his research there. Last fall, three groups — one at Delft, one in Austria, and the group at NIST-Boulder all conducted new tests, and all of them used Peter's protocol to prepare and conduct their tests. Every one reported that the statistical tests showed the experiments confirmed quantum mechanics, and the tests were loophole free, a remarkable result.
4. Meetings Organized:
    - The Workshop on Information and Processes held at CIAPA, Costa Rica (a Tulane meeting center). The WIP meeting was the latest in the WIP series, which began with Samson Abramsky's Clifford Lectures at Tulane in 2008. In the intervening time, there have been meetings at Tulane, at Dagstuhl, at CIAPA and this spring in Fontainebleau, France on the basic theme. The series attracts a somewhat electric group of researchers whose interests run the gamut from mathematics to computer science to various application areas, including systems biology and quantum information and computation. Details about the meetings can be found at http://cs.tulane.edu/~mwm/WIP2013/WIP_2013.html
    - The 29th Conference on the Mathematical Foundations of Programming Semantics, Tulane University, June, 2014. This annual series of conferences has been meeting since 1985, and the PI has been the principle organizer since the 1987 meeting. It publishes its proceedings in the Electronic Notes in Theoretical Computer Science, an open access online series that is published by Elsevier

that was founded by the PI in 1985; the PI is the Managing Editor of the series.
- The 30th Conference on the Mathematical Foundations of Programming Semantics, Cornell University, June, 2014.
- The 31st Conference on the Mathematical Foundations of Programming Semantics, Radboud University, Nijmegen, The Netherlands, June, 2015.

5. Honors: Editor, Semantics Column, SIGLOG Newsletter. SIGLOG is a new ACM special interest group formed in 2014. The PI was invited to edit a column on semantics for the newsletter.

Publications

1. MISLOVE, M. W. (2013). Anatomy of a Domain of Continuous Random Variables I. Theoretical Computer Science, to appear.
2. MISLOVE, M. W. (2013). Anatomy of a Domain of Continuous Random Variables II. Lecture Notes in Computer Science, 7860, 225-245.
3. MISLOVE, M. W., Kozen, D., editors, (2013). In Dexter Kozen and Michael Mislove (Ed.), Proceedings of the 29th Conference on Mathematical Foundations of Programming Semantics (MFPS 2013) (pp. 440), . Amsterdam: Elsevier Science. http://www.sciencedirect.com/science/journal/15710661
4. MISLOVE, M. W. (2014). Anatomy of a Domain of Continuous Random Variables I, Theoretical Computer Science, 546, 176-187. http://www.sciencedirect.com/science/article/pii/S0304397514001832
5. MISLOVE, M. W., Brian, W. R. (2014). From Haar to Lebesgue via Domain Theory, In Elham Kashefi, Catuscia Palamidessi, Jan Rutten and Alexandra Silva (Ed.), Festschrift for Prakash Panangaden's 60th Birthday (vol. 8464, pp. 214 - 228). Berlin, Heidelberg, New York: Lecture Notes in Computer Science.http://link.springer.com/chapter/10.1007%2F978-3-319-06880-0_11#page-1
6. BIERHORST, P., A rigorous analysis of the Clausner-Horne-Shimony-Holt inequality when trials need not be independent, Foundations of Physics 44, 736—761 (2014).
7. BIERHORST, P., A new loophole in recent Bell test experiments, arXiv:1311.4488, (2014).
8. BIERHORST, P., A Mathematical Foundation for Locality, Tulane University PhD Thesis, August, 2014.
9. BARKER, T., A monad for randomized algorithms, Tulane University PhD Thesis, May, 2016.
10. MISLOVE, M. W., Random Variables and Domain Theory, in preparation.
11. MISLOVE, M. W., A new monad of random variables, in preparation.

Invited Talks

1. MISLOVE, M. W., Workshop on Information and Processes, Tulane University, CIAPA, Costa Rica, "The Search for Random Variable Monads," International, Invited. (December 18, 2013).
2. BIERHORST, P., Random walks and Bell statistics, Tulane University, CIAPA, Costa Rica, "The Search for Random Variable Monads," International, Invited. (December 18, 2013).
3. MISLOVE, M. W., International Symposium on Domain Theory and Its Applications, Hunan University, China, Hunan University, Changsha, China, "Random Variable Models of Domains," Keynote Address. (October 28, 2013).
4. MISLOVE, M. W., Samson@60 Fest, Oxford University, Oxford, UK, "Random Variable Models of Computation," International, Refereed, Invited. (May 28, 2013).
5. MISLOVE, M. W., Domain XI - Annual Domains Workshop, University Denis Diderot,

Paris, France, "Cantor groups, Haar Measure and Lebesgue Measure on [0,1]," International workshop (September 9, 2014).

6. MISLOVE, M. W., PrakashFest - Workshop Honoring Prakash Panangaden on his 60th Birthday, University of Oxford, University of Oxford, England, "From Haar to Lebesgue via Domain Theory," International conference. (May 23, 2014).
7. MISLOVE, M. W., Domain XI - Annual Domains Workshop, University Denis Diderot, University Denis Diderot, Paris, France, "Cantor groups, Haar Measure and Lebesgue Measure on [0,1]," International, published in local proceedings, Accepted. (September 9, 2014).
8. MISLOVE, M. W., Domains XII - Annual Domains Workshop, “Another Random Variable Monad, Preliminary Report,” Part of Boole Bicentennial, University College Cork, Ireland, (August, 2015)
9. MISLOVE, M. W., New York Topology Seminar, CUNY, Queensboro Campus, “From Haar to Lebesgue via Domain Theory, October, 2015.
10. MISLOVE, M. W., Skorohod’s representation theorem and domain theory, Workshop on Information and Processes, Fontainebleau, France, April, 2016.

1.

1. Report Type

Final Report

Primary Contact E-mail
Contact email if there is a problem with the report.

mislove@tulane.edu

Primary Contact Phone Number
Contact phone number if there is a problem with the report

5048655803

Organization / Institution name

Tulane University

Grant/Contract Title
The full title of the funded effort.

A Mathematical Theory of System Information Flow,

Grant/Contract Number
AFOSR assigned control number. It must begin with "FA9550" or "F49620" or "FA2386".

FA9550-13-1-0135

Principal Investigator Name
The full name of the principal investigator on the grant or contract.

Michael Mislove

Program Manager
The AFOSR Program Manager currently assigned to the award

Dr. Trristan Nguyen

Reporting Period Start Date

03/27/2013

Reporting Period End Date

03/31/2016

Abstract

The initial goal of this project was to study information flow in computational systems using techniques from information theory, domain theory and other areas of mathematics and computer science. Over time, the focus shifted toward a better understanding of random variables, in particular from a domain-theoretic perspective. The research focused more narrowly on the relationship between random variables, domain theory and related work on information theory. The results produced by the project include two new models for probabilistic computation. one of which models randomized algorithms and the other of which is suitable for analyzing crypto-protocols. The project also produced a new statistical testing regimen for evaluating empirical tests of quantum phenomena that have since been the basis for validating the first loophole free tests of nonlocality. Finally. the project applied Skorohod's Theorem to analyze the domain structure of certain domains, and conversely, produced a domain-theoretic proof of Skorohod's Theorem that also shows every measure on countably-based coherent domains is the image of Haar measure on the Cantor set under a Scott-continuous mapping.

Distribution Statement
This is block 12 on the SF298 form.

Distribution A - Approved for Public Release

Explanation for Distribution Statement

If this is not approved for public release, please provide a short explanation. E.g., contains proprietary information.

SF298 Form

Please attach your SF298 form. A blank SF298 can be found here. Please do not password protect or secure the PDF The maximum file size for an SF298 is 50MB.

AFD-070820-035 copy.pdf

Upload the Report Document. File must be a PDF. Please do not password protect or secure the PDF . The maximum file size for the Report Document is 50MB.

final.pdf

Upload a Report Document, if any. The maximum file size for the Report Document is 50MB.

Archival Publications (published) during reporting period:

1. MISLOVE, M. W. (2013). Anatomy of a Domain of Continuous Random Variables I. Theoretical Computer Science, to appear.
2. MISLOVE, M. W. (2013). Anatomy of a Domain of Continuous Random Variables II. Lecture Notes in Computer Science, 7860, 225-245.
3. MISLOVE, M. W., Kozen, D., editors, (2013). In Dexter Kozen and Michael Mislove (Ed.), Proceedings of the 29th Conference on Mathematical Foundations of Programming Semantics (MFPS 2013) (pp. 440), . Amsterdam: Elsevier Science. http://www.sciencedirect.com/science/journal/15710661
4. MISLOVE, M. W. (2014). Anatomy of a Domain of Continuous Random Variables I, Theoretical Computer Science, 546, 176-187. http://www.sciencedirect.com/science/article/pii/S0304397514001832
5. MISLOVE, M. W., Brian, W. R. (2014). From Haar to Lebesgue via Domain Theory, In Elham Kashefi, Catuscia Palamidessi, Jan Rutten and Alexandra Silva (Ed.), Festschrift for Prakash Panangaden's 60th Birthday (vol. 8464, pp. 214 - 228). Berlin, Heidelberg, New York: Lecture Notes in Computer Science.http://link.springer.com/chapter/10.1007%2F978-3-319-06880-0_11#page-1
6. BIERHORST, P., A rigorous analysis of the Clausner-Horne-Shimony-Holt inequality when trials need not be independent, Foundations of Physics 44, 736—761 (2014).
7. BIERHORST, P., A new loophole in recent Bell test experiments, arXiv:1311.4488, (2014).
8. BIERHORST, P., A Mathematical Foundation for Locality, Tulane University PhD Thesis, August, 2014.
9. BARKER, T., A monad for randomized algorithms, Tulane University PhD Thesis, May, 2016.

2. New discoveries, inventions, or patent disclosures:

Do you have any discoveries, inventions, or patent disclosures to report for this period?

Yes

Please describe and include any notable dates

1. Domain-theoretic proof of Skorohod's Theorem, Spring, 2015
2. Development of loophole-free test of quantum versus alternative theories excluding nonlocality, Summer, 2014.
3. Development of monad for randomized algorithms, completed Spring, 2016
4. Proof that the sub-probability measures on a complete chain form a continuous lattice, Spring, 2016.
5. Development of monad for multiple participants in a computational process, ongoing.

Do you plan to pursue a claim for personal or organizational intellectual property?

No

Changes in research objectives (if any):

The original objective was to study informtiion flow in channels with varying arities, but the focus shifted a developing a better understanding of random variables from a domain-theoretic perspective.

Change in AFOSR Program Manager, if any:

The project was originally funded by Dr. Robert Bonneau, but was transferred to Dr. Tristan Nguyen when Dr. Bonneau left AFOSR.

Extensions granted or milestones slipped, if any:

AFOSR LRIR Number

LRIR Title

Reporting Period

Laboratory Task Manager

Program Officer

Research Objectives

Technical Summary

Funding Summary by Cost Category (by FY, $K)

	Starting FY	FY+1	FY+2
Salary			
Equipment/Facilities			
Supplies			
Total			

Report Document

Report Document - Text Analysis

Report Document - Text Analysis

Appendix Documents

2. Thank You

E-mail user

Jun 17, 2016 21:41:09 Success: Email Sent to: mislove@tulane.edu